485. Proposed by NATHAN ALTSHILLER, University of Colorado.

Find the surface generated by the orthogonal projection of a given line upon a variable plane turning about a fixed axis.

CALCULUS.

403. Proposed by C. N. SCHMALL, New York City.

A paraboloid of revolution generated by the curve $x^2 = 4ay$, contains a quantity of water such that if a sphere of radius r be dropped to the bottom, it will just be covered by the water. Show that if the volume of water used in this experiment is to be a *minimum*, then we must have $a = r/6$.

404. Proposed by B. J. BROWN, Victor, Colorado.

Solve the differential equation, $(x^2 - y^2)(1 + dy/dx) = 2xy(1 - dy/dx)$.

MECHANICS.

321. Proposed by E. J. MOULTON, Northwestern University.

The attraction, A, in any given direction, due to a homogeneous sphere, on a particle at the center of the sphere, using the Newtonian law, is obviously zero. Find the error in the following method of computing A. Take cylindrical coördinates with origin at the center of the sphere; let the z-axis extend in the direction of the attraction to be computed, and let r, θ be the polar coördinates used. Let δ be the density and R the radius of the sphere, and k the constant of gravitation. Then

$$A = \int_{z=-R}^{z=R} \int_{r=0}^{r=\sqrt{R^2-z^2}} \int_{\theta=0}^{\theta=2\pi} \frac{k\delta rz d\theta dr dz}{[r^2 + z]^{3/2}} \tag{1}$$

$$= 2\pi k\delta \int_{z=-R}^{z=R} \left[\frac{-z}{(r^2 + z^2)^{1/2}} \right]_{r=0}^{r=\sqrt{R^2-z^2}} dz \tag{2}$$

$$= 2\pi k\delta \int_{-R}^{R} \left[\frac{-z}{R} + 1 \right] dz \tag{3}$$

$$= 4\pi k\delta R. \tag{4}$$

322. Proposed by FRANK R. MORRIS, Glendale, Calif.

A pole of uniform size and weight throughout its length stands in a vertical position. The height of the pole is h and weight w. It hinges at the base and falls, passing through a horizontal position. At the moment it reaches the horizontal position, how far from the base is the maximum vertical force tending to break the pole? How great is this force? What is the horizontal force at the same position in the pole?

323. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

Two equal bodies are placed on a rough inclined plane, being connected by a light inelastic string; if the coefficients of friction are respectively $1/3$ and $1/4$, show that they will both be on the point of motion when the inclination of the plane is $\sin^{-1}(7/25)$.

NUMBER THEORY.

239. Proposed by HAROLD T. DAVIS, Colorado Springs, Colorado.

Give a general method for determining the solution in integers of the equation

$$x^r - 10xy - (n + 1) + y = 0,$$

where r and n are positive integers.

240. Proposed by J. W. NICHOLSON, Louisiana State University.

If the roots of $x^3 - px + q = 0$ are rational, prove that $4p - 3x^2$ is a perfect square.

241. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

If $a^2 + b^2 = c^2$, where a, b, and c are integers, then prove that abc will be a multiple of 60.

In the next issue we shall reprint all unsolved problems in Number Theory published since January, 1913. They are numbers 191, 192, 196, 198, 201, 202, 205, 208, 209, 211, 214, 217, 219, 221, 222, 223. Please have these in mind. EDITORS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

433. Proposed by B. J. BROWN, Student at Drury College.

Prove that, if all the quantities, a, b, etc., are real, then all the roots of the equati ns

$$\begin{vmatrix} a-x & h \\ h & b-x \end{vmatrix} = 0, \qquad \begin{vmatrix} a-x & h & g \\ h & l-x & f \\ g & f & c-x \end{vmatrix} = 0$$

are real; and generalize the proposition.

SOLUTION BY WM. E. HEAL, Washington, D. C.

The first equation may be written

$$\left[x - \left(\frac{a+b}{2}\right)\right]^2 = \frac{(a-b)^2}{4} + h^2.$$

Since the second member is the sum of two squares and so can never become negative, if a, b, and h are real, it follows that both roots are real.

The second equation is proved, in Salmon's *Modern Higher Algebra*, 4th edition, page 28, to have its roots all real.

The general equation, of which the above are special cases, is shown on page 48 of the same work to have all its roots real.

Thus also referred to by A. M. HARDING.

442. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

Show that the sum of n terms of the series $1/2 - 1/3 + 1/4 - 1/6 + 1/8 - 1/12 + \cdots$ is $1/3[1 - (1/2)^{n/2}]$ when n is even, and $1/3[1 + 2\sqrt{2}(1/2)^{(n/2)+1}]$ when n is odd.

SOLUTION BY IRBY C. NICHOLS, Chicago, Ill.

(1) *When n is even.* Grouping the terms successively by twos, we have a series of $n/2$ terms from which the factor $(\frac{1}{2} - \frac{1}{3})$ may be removed, thus,

$$(\tfrac{1}{2} - \tfrac{1}{3})\left[\frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \cdots \frac{1}{2^{n/2-1}}\right].$$

The series in brackets is geometrical, and we have the sum

$$(\tfrac{1}{2} - \tfrac{1}{3})\left[\frac{1 - \frac{1}{2}\cdot\frac{1}{2^{n/2-1}}}{\frac{1}{2}}\right] = \tfrac{1}{3}[1 - (\tfrac{1}{2})^{n/2}].$$

(2) *When n is odd.* Then the sum of $n+1$ terms can be written by (1), using $n+1$ for n. If now we add the $(n+1)$th term to this sum, we shall have the sum of n terms, since the $(n+1)$th term is negative. This gives

$$\begin{aligned} S_n &= \tfrac{1}{3}[1 - (\tfrac{1}{2})^{(n+1)/2}] + (\tfrac{1}{2})^{[(n+1)/2]-1} \\ &= \tfrac{1}{3}[1 - 2^{1/2}(\tfrac{1}{2})^{n/2+1}] + 2^{3/2}(\tfrac{1}{2})^{n/2+1} \\ &= \tfrac{1}{3}[1 + \sqrt{2}(\tfrac{1}{2})^{n/2+1}]. \end{aligned}$$